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Kronecker factorization theorems for the exceptional Malcev algebra

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In this paper, we prove that a Malcev algebra $\mathbb{M}$ that belongs to a certain variety of Malcev algebras $\mathbb{H}$ such that $\mathbb{M}$ contains the seven-dimensional exceptional Malcev algebra $\mathbb{M}$ is isomorphic to a direct sum $(\mathbb{M} \otimes_F A) \oplus \mathbb{N}$, where A is an associative commutative algebra. Also, we prove that a Malcev superalgebra $\mathbb{M} = \mathbb{M}_0 \oplus \mathbb{M}_1$ in $\mathbb{H}$ whose even part $\mathbb{M}_0$ contains $\mathbb{M}$ is isomorphic to a direct sum $(\mathbb{M} \otimes_F \mathrm{A}) \oplus \mathbb{N}'$, where A is an associative supercommutative superalgebra.

Keywords: Malcev (super)algebra; non-Lie Malcev algebra; Malcev module; Kronecker factorization theorem.

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1. Introduction

A *Malcev algebra* is a vector space M with a bilinear binary operation $(x, y) \mapsto xy$ that satisfies the following identities:

$$x^2 = 0, \quad J(x, y, xz) = J(x, y, z)x, \tag{1}$$

where $J(x, y, z) = (xy)z + (yz)x + (zx)y$ is the Jacobian of the elements $x, y, z \in M$. Denote by $J(M, M, M)$ the subspace spanned by the Jacobians, which is an ideal of M (see [23]). The set $N_M = \{x \in M : J(x, M, M) = 0\}$ is also an ideal of M called the *Lie nucleus* of M. A Malcev algebra M is a Lie algebra if and only if $J(M, M, M) = 0$ if and only if $N_M = M$. In some way, Malcev algebras with a trivial Lie nucleus, $N_M = 0$, are far from being Lie algebras.

All Lie algebras are clearly Malcev algebras because the Jacobian of any three elements vanishes. The tangent space $T(L)$ of an analytic Moufang loop L is another